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Improvement of The Buffered Storage Assignment Technique for The Packets Switch Node

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Abstract

Consider the packets switch node's buffer, which is shared by numerous output communications lines. Spreading buffered memory across numerous users minimizes the total amount of storage required to fulfill latencies constraints and the possibility of packets loss. despite this, there can be an issue with assigning buffers memory amongst various users given that specific users who have consumed most overall that memory could limit or restrict accessibility to communications connections for others users, considerably reducing the overall efficiency of the switch node. These are several buffers storage allocation strategies, among which, known as SMA (Share in Minimum Assignment), are being studied in this research to decrease the expenses related to packets denial and postponement, as well as the functioning of the drives and lines of communication. The switch nodes are modelled using a multithreaded queue system with parallel devices of the kind, a memory buffer sharing accordingly to the SMA scheming, and a set amount of memory spaces designated to every device. A mathematical description of the problems of optimising the SMA schemes in regard to the amount of publically available buffer placements is presented with the goal to reduce losses to the system caused by applications disapproval, application queuing latency, and buffers and device operation. The hypothesis about the bounds of the field that contains the global optimal point is proven. A variety of arguments are also provided as a result of the theorem regarding the location of the globally optimal of the function of objective for different switch nodes types and specific instances of SMA.

A. Introduction

In packet-switched communication networks, have in which packets arriving from all input communication lines are stored in anticipation before being transmitted to the corresponding outputs. The quality of service in such networks largely depends on the memory capacity of the management system. This dependence arises due to the fact that with limited bandwidth of communication channels and a small amount of memory, there may be unacceptably large packet losses or an increase in the number of packet retransmissions and, as a result, an unacceptably strong decrease in network performance, and with a large amount of memory, long queues are possible and, as a consequence, unacceptably long delays of packets in the management system. Sharing shared memory in such a multi-user system can significantly improve the performance of the management system, but creates a new problem - it requires power supply management [1]. Hence the problem arises of choosing the optimal volume and distribution scheme of the management system in order to meet certain requirements for the quality of packet service and costs associated with the cost of equipment and technical operation of the management system [1].

Immediately after the appearance of communication networks with several works were published that considered the problem of sharing. For example,[2] in a buffer memory sharing scheme was proposed, which uses a threshold rule close to the optimal one, and in [3] various sharing plans were investigated, that is: (Complete Sharing), where an incoming package is acknowledged, if any space for storage is given. (Complete Partitioning), so that all the storage is constantly distributed between output lines; Sharing with Maximum Assignment, which limits the number of storage locations, dedicated to each output line; SMA, a minimal amount of storage spaces are Always kept for every output line, as well as the rest are equally available to all output lines; (Sharing with Maximum Logjam Distance and Minimal Assignment), It is a mix of Sharing with Maximum Assignment and SMA schemes. Along with choosing an effective scheme, it is also important to optimize the parameters of this scheme, in particular the total amount of memory, which, with the exception of isolated special cases (some of them are referenced below), remains an open problem (in the sense of its exact solution) due to its complexity. As a rule, it is reasonable to expect an increase in throughput (here and below, throughput is defined as the intensity of the packet flow at the output of the system) with increasing buffer capacity. This has been verified for many standard, such as the queue (see, for example, [4, 5]), and it was shown in [6] that the finding is also valid underneath more broad requirements. However, in general, the outcome is incorrect (see, for example, [7]).

Researchers are still showing interest in managing power supply, as can be seen from the many scientific articles in domestic and foreign scientific journals published recently (see, for example, [8]). They are mainly devoted to the analysis of various power supply distribution schemes, but there are few works in the literature in which exact solutions for optimizing the schemes are obtained. Below is an overview of some of them, in which studies were carried out on the dependence of quality indicators of the management company on the type of power supply distribution scheme and the values of its parameters. In [8], they consider the the line, generate equations for several performance indicators relating to velocity and averaged bit rates, and present some structure results about the relationships between these metrics. Changes in buffer capacity have been shown to have the most significant impact when the method load is additionally very small nor excessively large. For the system, it is shown that the increase in throughput (both nominal and relative) that would be obtained by adding additional buffer space is unimodal in load. In [9], a heuristic algorithm for solving the problem of joint control of the volume of power supplies and the transmission speed of communication lines is proposed, which is considered in the following aspects:

(1) For a given degree of channel load and a given transmission speed, find the required volume of buffer memory switching node that provides the given average delay in the network and the required probability of cell loss due to buffer overflow;

(2) For a given degree of channel load and known buffer memory volumes, find the required transmission speed that provides the given average network delay and the required probability of cell loss due to buffer overflow.

In [10], they solved the problem of selecting the volume of a storage unit for a buffer memory of type, formulating it as a nonlinear programming problem:

(1) For a fixed input load, find the volume of a storage device (buffer memory) at which the average delay reaches a minimum, and the intensity of losses does not exceed a given value;

(2) For a fixed storage volume (buffer memory), find the value of the input load at which the average delay reaches a minimum and the loss rate does not exceed a specified value.

In [11], under the assumption of poison incoming flows, exponential service time and single-channel transmission lines, the dynamic distribution (control) of the power supply was considered and, for the case of three channels, the form of the admissible state space corresponding to the optimal solution was obtained. A dynamic buffer memory-sharing scheme that it is proposed that each queue can expand to a dynamically determined threshold. [12].This criterion is derived as the intersection of the leftover buffer and a predetermined value. [7] Proposed a sharing scheme (called drop on demand) that allows received packets to be dropped and, therefore, does not belong to the class of schemes mentioned above. According to this scheme, an incoming packages are usually allowed if the buffer is empty. If a package meant for outputs line (i) arrive and a buffer is discovered to be full, and outputs line (I) has additional packages in sharing memory then each another port, the that follows action is performed:

– If i = I, then the incoming packet is discarded;

– If i ≠I, then the incoming packet is received in the buffer and one packet to line (I) is discarded.

Numerical examples have been given to show that the drop on demand scheme provides better throughput than the Complete Sharing and Complete Partitioning schemes. In [13], using the example of a two-channel, it is shown this strategy is only optimum for symmetrical systems. It is also stated there that dynamic power supply distribution schemes are difficult to implement compared to static ones. For a finite shared buffer memory and several packet queues, [14] considers a scheme with dynamic individual ceilings depending on the number of free seats in the buffer memory (called - Flow Attentive Buffer). The results of a numerical comparative analysis of the schemes of full sharing, full sharing, dynamic sharing and Flow

Attentive Buffer are presented using examples of the switching node model in the form of parallel queues with limited storage. In [15], the problem of optimizing the total cost costs (customer loss cost, buffer storage cost and operating costs) of the type with several releases was formulated and the influence of system parameters such as the buffer size, release duration and service time distribution was numerically studied , on performance indicators and overall cost. An algorithm for optimizing the scheme with an objective income function that takes into account the average delay and the probability of packet loss was proposed in [16] within the framework of a management system model represented by parallel systems of type. Within the framework of this model, the unimodality of the objective function in terms of the volume of the switching node has been proven. A similar statement for the circuit has been proven in the case of a control system represented by the parallel of type with backup channels [15]. Below we study the problem of optimizing the SMA scheme with a cost objective function that takes into account the costs due to packet losses, delay of packets in the queue and operation of the drive and devices.

B. Switching node model and problem statement

As a management system model, we consider a multi-threaded and multi-channel with a common storage unit (buffer memory) of capacity N, which receives n Poisson flows of requests (packets) with intensities $\lambda j > 0, j = 1, \ldots, n,$ in which to each j_{th} flow for servicing requests a corresponding j_{th} line of s_j similar devices (channels) is attached. In this case, each request can occupy only one free space (cell) in the storage, and only requests from the j-flow are received on the j-line. Let us introduce the following notation: k_j is the number of j-claims in the storage, $\bar{k} = (k_1, \ldots k_n)$ is the system state vector; $\bar{a} = (a_1, \ldots a_n), a_j \geq 0$, is the number of places assigned to j-requests in the accumulator (which can only be occupied by j-requests),$\sum_{j=1}^{n} a_j \leq N; L -$ number of publicly available spaces in the drive:

$$L = N - \sum_{j=1}^{n} a_j;\ K^L = \{\bar{k}: \sum_{j=1}^{n} k_j \ \leq \sum_{j=1}^{n} a_j; + L$$

— the set of all possible states of the system; $\bar{K}_j^L = \{\bar{k} \in K^L : k_j \geq a_j\ and\ \sum_{j=1}^{n} \left(k_j - a_j\right) = L$

a set of states in which the j-request is not allowed into the storage (it is lost); $\bar{K}_j^L = \{\bar{k} \in K^L : k_j \geq a_j\ or\ \sum_{j=1}^{n} \left((k_j - a_j)^+\right) < L$

a number of states in which a j-request has available space in the storage, where

$$(k_j - a_j)^+ = \begin{cases} k_j - a_j & , k_j > a_j\ ; \\ 0 & , \ k_j \leq a_j\ . \end{cases}$$

According to the introduced notation, applications are admitted to the accumulator according to the scheme [3]: a j-application is admitted to the accumulator if the condition $\bar{k} \in K_j^L$ is met, and $\bar{k} \in \bar{K}_j^L$ is rejected (lost). Each incoming j-request occupies one of the available and free places in the storage and one free device in the j-line immediately, if $k_j < s_j$, or after release, if $k_j \geq s_j$, and after completion of service leaves the system, freeing up the device and space at the same time in the drive. The service time of a j-request is an exponential random variable with a given parameter $\mu j, j = 1, \ldots, n$.

Let us assume that j-requests first occupy the places assigned to them and, if there is a public place occupied by a j-request, then the assigned place vacated by the j-request becomes public, and this occupied public place becomes assigned. Thus, the number of seats assigned to each stream remains constant. If a newly received request finds all the available storage spaces occupied, it is lost forever. The process of transition of the described QS from state to state and has the following distribution of stationary state probabilities [5]:

$$\pi_{\bar{k}}(L) = \pi_{\bar{0}}(L) \prod_{j=1}^{n} z_j(k_j). \qquad (1)$$

Here

$$\pi_{\bar{0}}(L) = [\sum_{k \in K} \prod_{j=1}^{n} z_j(k_j)]^{-1}, \bar{0} = \{\bar{k}: k_i = 0, i = 1, \ldots, n\};$$

$$z_j(k_j) = \begin{cases} \dfrac{p_j^{k_j}}{k_j!}, & 0 \le k_j \le s_j; \\ \dfrac{p_j^{s_j}}{s_j!}\left(\dfrac{p_j}{s_j}\right), & s_j \le k_j \le a_j + L, \end{cases}$$

Where

$$p_j = \frac{\lambda_j}{\mu_j}.$$

In what follows, we consider the system only in a stationary operating mode and assume that the values of the parameters $a_j = s_j$, $j = 1, \ldots, n$, and are constant quantities.

As an indicator of system efficiency, the marginal revenue function per unit of time is used, which takes into account service fees, losses due to waiting in line, rejected applications and system maintenance. It is believed that the system receives a service fee at the time the application is received into the storage facility. Income is measured in cost units and also depends on the following cost parameters:

$C_{0,i} \ge 0$ payment received by the system if the received i-request is serviced by the system; $C_{1,i} \ge 0$ penalty for rejecting the received i-application; $C_{2,i} \ge 0$ penalty per unit of waiting time for the i-request in the queue; $C_3 \ge 0$ system costs per unit of time for maintenance of one storage location; $C_4 \ge 0$ system costs per unit time for maintenance of all system devices.

Income is expressed by a function of the following form:

$$Q(L) = \lambda \sum_{\bar{k} \in K} \pi_{\bar{k}}(L) q_{\bar{k}}, \lambda = \sum_{j=1}^{n} \lambda_j$$

Where L is a variable value; $q_{\bar{k}}$ is the average income received by the system for the period between adjacent moments of receipt of requests from outside, if at the moment of receipt at the beginning of the period the system was in state $\bar{k}$.

The problem of optimizing the volume of publicly accessible storage spaces is formulated as a mathematical problem

$$L^* = agr\max_{L \ge 0} Q(L).$$

C. Result and Discussion

Let us introduce the following notation:

$\overline{K}_{jm}^{L} = \{\bar{k} \in \overline{K}_{j}^{L}: k_j m \geq a_j, \sum_{i=1, i\neq j}^{n}(k_i - a_j)^{+} = L - (m - a_j)\}$;

$$K_{jm}^{L} = \left\{\bar{k} \in \overline{K}_{j}^{L}: k_j m, \sum_{i=1, i\neq j}^{n} (k_i - a_j)^{+} \ or \ k_j = m < a_j\right\};$$

$\pi_{j,m}^{-}(L) = \sum_{\bar{k}\in K_{jm}^{L}} \pi_{\bar{k}}(L)$ -is the stationary probability that $\bar{k} \in K_{jm}^{L}$; $\pi_{j,m}^{+}(L) = \sum_{\bar{k}\in K_{jm}^{L}} \pi_{\bar{k}}(L)$ stationary probability that is the stationary probability that $k_j = m; x_i(\bar{k})$

Heaviside function:

$$x_j(\bar{k}) = \begin{cases} 1, & \bar{k} \in K_j^L; \\ 0, & \bar{k} \in \overline{K}_j^L; \end{cases}$$

$d_{j,m}$ is the cost of the average total losses due to waiting and rejection of j-applications for the period between adjacent moments of receipt of external applications, if at the beginning of the period the number of j-applications in the accumulator was equal to m.

Theorem 1. The function Q(L) can be written as:

$Q(L) = \sum_{j=1}^{n}\left[(\lambda - \lambda_j)Q_j^{+}(L) + \lambda_j Q_j^{-}(L)\right] - C_3 N - C_4$, (2)

where $Q_j^{+}(L)$ *and* $Q_j^{-}(L)$, $j = 1, \dots n$, — functions unimodal with respect to $L \geq 0$:

$$Q_j^{+}(L) = \sum_{m=0}^{a_j+L-1} d_{j,m}\pi_{j,m}^{-}(L) + \sum_{m=0}^{a_j+L} d_{j,m}\pi_{j,m}^{+}(L);$$

$$Q_j^{-}(L) = \sum_{m=0}^{a_j+L-1} (d_{j,m+1} + C_{0,j})\pi_{j,m}^{-}(L) + \sum_{m=0}^{a_j+L} (d_{j,m} - C_{1,j})\pi_{j,m}^{+}(L).$$

Proof. It is easy to see that for income $q_{\bar{k}}$ and values $d_{j,m}$, $j = 1, \dots, n, m = s_j - 1, \dots, a_j + L$ the following relation holds:

$$q_{\bar{k}}^{-} = \begin{cases} \sum_{l=1, l\neq j}^{n} d_{l,k_l} + d_{j,m+1}C_{0,j} - \frac{C_3 N + C_4}{\lambda}, \\ \quad if\ a\ j-order\ is\ received\ from\ outside\ \ and\ \bar{k} \in K_{j,m}^{l}; \\ \sum_{l=1}^{n} d_{l,k_l} - C_{1,j} - \frac{C_3 N + C_4}{\lambda}, \\ \quad if\ a\ j-order\ \ and\ k\ are\ received\ from\ outside\ \ and\ \bar{k} \in \overline{K}_{j,m}^{l}. \end{cases}$$

Then the following expression is valid for the objective function:

$Q(L) = \lambda \sum_{\bar{k}\in K^L} \pi_{\bar{k}}(L) \sum_{j=1}^{n} \frac{\lambda_j}{\lambda} [\sum_{l=i, l\neq j}^{n} d_{l,k_l} + \left(d_{j,k_{j+1}} + C_{0,j}\right) x_j(\bar{k}) + \left(d_{j,k_{j+1}} + C_{1,j}\right)\left[1 - x_j(\bar{k})\right] - \frac{C_3 N + C_4}{\lambda}] = \sum_{j=1}^{n} \lambda_j [\sum_{l=1}^{n} \sum_{m=0}^{a_l+L} \pi_{l,m}(L) d_{l,m} - \sum_{m=0}^{a_l+L} \pi_{l,m}(L) d_{l,m} + \sum_{m=0}^{a_l+L-1} \pi_{j,m}^{-}(L) d_{l,m+1} + \sum_{m=0}^{a_l+L} \pi_{j,m}^{+}(L) d_{l,m} +$

$C_{0,j}\sum_{m=0}^{a_l+L-1}\pi_{j,m}^{-}(L)-C_{1,j}\sum_{m=0}^{a_j+L}\pi_{j,m}^{+}(L)]-C_3N-C_4=\sum_{j=1}^{n}(\lambda-\lambda_j)[\sum_{m=0}^{a_l+L-1}\pi_{j,m}^{-}(L)d_{l,m+1}+\sum_{m=0}^{a_l+L}\pi_{j,m}^{+}(L)]+\sum_{j=1}^{n}\lambda_j[\sum_{m=0}^{a_j+L-1}(d_{j,m+1}C_{0,j})\pi_{j,m}^{-}(L)+\sum_{m=0}^{a_j+L}(d_{j,m}-C_{1,j})\pi_{j,m}^{+}(L)]-C_3N-C_4$.

Therefore, equality (2) is satisfied. Let us present analytical expressions for the parameter $d_{j,m}$. Let us use the results of work [17]. According to this work (see the derivation of formula (5) in [10]),

$$d_{j,m}=\begin{cases}0, m\le s_j;\\ -\frac{C_{2,j}}{2\mu_j s_j}[\sum_{l=1}^{m+1-s_j}l(l+2s_j-2m-1)r_{j,l}-(m-s_j)(m+1-s_j)\sum_{l=m+2+s_j}^{\infty}r_{j,l}],\ s_j\le m\le a_j+L;\\ d_{j,m-1}, m=a_j+L,\end{cases}$$

Where $r_{j,l}$ is the probability that during the period between adjacent arrivals of applications for j-lines they will complete servicing exactly l applications, provided that at the beginning of the period there are at least l applications in the queue:

$$r_{j,l}=\lambda\int_0^{\infty}\frac{(\mu_j s_j t_j)^l}{l!}e^{-(\mu_j s_j+\lambda)t}dt\ \ ,l\ge 0.$$

According to work [10], the following relation is also valid:

$$d_{j,m}=d_{j,m+1}-\frac{C_{2,j}}{\mu_j s_j}\sum_{l=1}^{m+1-s}l r_{j,l}-\frac{C_{2,j}(m+1-s_j)}{\mu_j s_j}\sum_{l=m+2-s_j}^{\infty}r_{j,l}\ ,s_j\le m\le a_j+L-1. \quad (3)$$

Lemma 1. For probabilities $\pi_{j,m}^{-}(L) and \pi_{j,m}^{+}(L), j=1,\dots,n, m,=s_j-1,\dots,a_j+L-1$, the equalities are valid:

$$\left.\begin{aligned}\pi_{m+1}^{-}(L+1)=\pi_{j,m}^{-}(L)A_j(L+1);\\ \pi_{m+1}^{+}(L+1)=\pi_{j,m}^{+}(L)A_j(L+1),\end{aligned}\right\}\ (4)$$

Where

$$A_j(L+1)=\frac{1-P_{j,s_{j-1}}(L+1)}{1-P_{j,s_{j-2}}(L+1)}, P_{j,m}(L)=\sum_{l=0}^{m}\pi_{\ l,j}(L).$$

Proof . From (1) and equality $K_{j,m}^{L}=K_{j,m+1}^{L+1}$ should

$$\pi_{m+1}^{-}(L+1)-\pi_{m}^{-}(L)=\pi_0^{-}(L+1)\frac{P_j^{s_j}}{s_j!}\left(\frac{P_j}{s_j}\right)^{m+1-s_j}\sum_{\bar{k}\in K_{j,m+1}^{L+1}}\prod_{l=1,l\ne j}^{n}Z_j(k_j)-\pi_0^{-}(L)\frac{P_j^{s_j}}{s_j!}(\frac{P_j}{s_j})^{m-s_j}\sum_{\bar{k}\in K_{j,m}^{L}}\prod_{l=1,l\ne j}^{n}Z_j(k_j)\times[\frac{P_j}{s_j}[\sum_{m=0}^{s_j-1}\frac{p_j^m}{m!}\sum_{\bar{k}\in K_{j,m+1}^{L+1}}\prod_{l=1,l\ne j}^{n}Z_j(k_j)-\frac{P_j^{s_j}}{s_j!}\sum_{m=0}^{s_j-1}\frac{P_j}{s_j}\Bigg]+\frac{P_j^{s_j}}{s_j!}\sum_{\bar{k}\in K_{j,m+1}^{L+1}}\prod_{l=1,l\ne j}^{n}Z_j(k_j)]$$

Therefore, the first equality in (4) is satisfied. The second equality in (4) is proved in exactly the same way.

Lemma 2. The functions $Q_j^+(L) and Q_j^-(L)$ satisfy the following relations:

$$\left.\begin{aligned} Q_j^-(L) - Q_j^-(L+1) = [1 - A_j(L+1)][Q_j^-(L) - G_j^-(L); \\ Q_j^+(L) - Q_j^+(L+1) = [1 - A_j(L+1)][Q_j^+(L) - G_j^+(L); \end{aligned}\right\} \quad (5)$$

Where

$$G_j^-(L) = C_{0,j} + \frac{C_{2,j}}{1 - A_j(L+1)}[\sum_{m=s_j}^{a_j+L}[\frac{1}{\lambda} - \frac{1}{\mu_j s_j}\sum_{l=m+2-s_j}^{\infty}(l - m - 1 + s_j)r_{j,l}]\pi_{j,m}^-(L+1) + \sum_{m=s_j}^{a_j+L+1}[\frac{1}{\lambda} - \frac{1}{\mu_j s_j}\sum_{l=m+1-s_j}^{\infty}(l - m + s_j)r_{j,l}]\pi_{j,m}^+(L+1)];$$

$$G_j^+(L) = \frac{C_{2,j}}{1 - A_j(L+1)}[\sum_{m=s_j}^{a_j+L}[\frac{1}{\lambda} - \frac{1}{\mu_j s_j}\sum_{l=m+1-s_j}^{\infty}(l - m - 1 + s_j)r_{j,l}]\pi_{j,m}^-(L+1) + \sum_{m=s_j}^{a_j+L+1}[\frac{1}{\lambda} - \frac{1}{\mu_j s_j}\sum_{l=m+1-s_j}^{\infty}(l - m + s_j)r_{j,l}]\pi_{j,m}^+(L+1)].$$

Proof . Let us prove the first equality in (5). Let us denote $\Delta_{j,m+1} = d_{j,m+1} - d_{j,m}$ From (2) and (4) after simple transformations we obtain the equalities:

$Q_j^-(L) - Q_j^-(L+1) = \sum_{m=0}^{s_j-2}(d_{j,m+1} + C_{0,j})\pi_{j,m}^-(L) - \sum_{m=0}^{s_j-1}(d_{j,m+1} + C_{0,j})\pi_{j,m}^-(L+1) + \sum_{m=s_j-1}^{a_j+L-1}(d_{j,m+1} + C_{0,j})\pi_{j,m}^-(L) - \sum_{m=s_j}^{a_j+L}(d_{j,m+1} + C_{0,j})\pi_{j,m}^-(L+1) + \sum_{m=0}^{s_j-2}(d_{j,m} + C_{1,j})\pi_{j,m}^+(L) - \sum_{m=0}^{s_j-1}(d_{j,m} + C_{1,j})\pi_{j,m}^+(L) + \sum_{m=s_j-1}^{a_j+L}(d_{j,m} + C_{1,j})\pi_{j,m}^+(L) - \sum_{m=s_j}^{a_j+L+1}(d_{j,m} + C_{1,j})\pi_{j,m}^+(L) = C_{0,j}[P_{j,s_j} - 2L - P_{j,s_{j-1}}(L+1) + \sum_{m=s_j-1}^{a_j+L-1}(d_{j,m+1} + C_{0,j})\pi_{j,m}^-(L) - A_j(L+1)\sum_{m=s_j-1}^{a_j+L-1}(d_{j,m+1} + \Delta_{j,m+1} + C_{1,j})\pi_{j,m}^-(L) - -A_j(L+1)\sum_{m=s_j-1}^{a_j+L}(d_{j,m+1} + \Delta_{j,m+1} + C_{1,j})\pi_{j,m}^-(L)$

Applying formula (3), we obtain

$Q_j^-(L) - Q_j^-(L+1) = [1 - A_j(L+1)][Q_j^-(L) - C_0 - \frac{C_{2,j}}{1-A_j(L+1)}[\sum_{m=s_j}^{a_j+L}[\frac{1}{\lambda} - \frac{1}{\mu_j s_j}\sum_{l=m+2-s_j}^{\infty}(l - m + s_j)r_{l,j}]\pi_{j,m}^-(L+1) + \sum_{m=s_j}^{a_j+L+1}[\frac{1}{\lambda} - \frac{1}{\mu_j s_j}\sum_{l=m+1-s_j}^{\infty}(l - m + s_j)r_{l,j}]\pi_{j,m}^+(L+1)]]$. (6)

Therefore, the first equality in (5) is satisfied. In the same way, for the second equality in (5) we get:

$Q_j^+(L) - Q_j^+(L+1) = [1 - A_j(L+1)][Q_j^+(L) - C_0 - \frac{C_{2,j}}{1-A_j(L+1)}[\sum_{m=s_j}^{a_j+L}[\frac{1}{\lambda} - \frac{1}{\mu_j s_j}\sum_{l=m+2-s_j}^{\infty}(l-m+s_j)r_{l,j}]\pi_{j,m}^{-}(L+1) + \sum_{m=s_j}^{a_j+L+1}[\frac{1}{\lambda} - \frac{1}{\mu_j s_j}\sum_{l=m+1-s_j}^{\infty}(l-m+s_j)r_{l,j}]\pi_{j,m}^{+}(L+1)]]$. (7)

Consequently, the second equality in (5) also holds.

Lemma 3. The functions $G_j^-(L)$ *and* $G_j^+(L)$ are non-increasing functions.

Proof . Let us note that the sum in the outer square bracket in (6) expresses the average time the j-line is in a state with fully occupied devices for the period between adjacent arrivals of applications in a system with publicly available places in the amount of L+1, provided that the one arriving from outside the claim belongs to the j-stream, and the sum in the outer square bracket in (7) expresses the same for the j-line provided that the received request does not belong to the j-flow. Therefore, obviously, the expressions in the indicated brackets are functions increasing in L.

Let's consider the difference

$A_j(L+2) - A_j(L+1) = \frac{1-P_{j,s_j-1}(L+2)}{1-P_{j,s_j-2}(L+1)} - \frac{1-P_{j,s_j-1}(L+1)}{1-P_{j,s_j-2}(L)}$. (8)

The following equalities are valid:

$\frac{\pi_0^-(L+1)}{\pi_0^-(L+2)}\left[1-P_{j,s_j-1}(L+2)\right]\left[1-P_{j,s_j-2}(L)\right] = \frac{\pi_0^-(L)}{\pi_0^-(L+1)}[1-P_{j,s_j-1}(L+1)]^2 = [\sum_{\bar{k}\in K_{j,s_j}^{L+1}\cup \bar{K}_{j,s_j}^{L+1}}\prod_{l=1,l\neq j}^{n} Z_j(k_j) + \sum_{m=s_j+1}^{a_j+L+2}(\frac{p_j}{s_j})^{m-s_j}\sum_{\bar{k}\in K_{j,m}^{L+1}\cup \bar{K}_{j,m}^{L+1}}\prod_{l=1,l\neq j}^{n} Z_j(k_j)] \times [\sum_{m=s_j+1}^{a_j+L+2}(\frac{p_j}{s_j})^{m-s_j}\sum_{\bar{k}\in K_{j,m}^{L+2}\cup \bar{K}_{j,m}^{L+2}}\prod_{l=1,l\neq j}^{n} Z_j(k_j)]$

It follows that the sign of the difference (8) coincides with the sign of the difference $\frac{\pi_0^-(L+2)}{\pi_0^-(L+1)} - \frac{\pi_0^-(L+1)}{\pi_0^-(L)}$.

For the system under consideration, in the case of a drive with (L + 1) public places, the value $\frac{\pi_0^-(L+1)}{\pi_0^-(L)}$ is the probability that the drive has at least one free place, and, obviously, this value increases with increasing L . It follows that the above difference for all $L \geq 0$ is a positive value. Therefore, $G_j^-(L)$ is a non-increasing function. The lemma is proved in exactly the same way in the case of the function $G_j^+(L)$. From Lemmas 1 and 2 it follows that the functions $Q_j^-(L)$*and* $Q_j^+(L)$ satisfy all the conditions of the theorem in [18, 19], which implies the validity of the statements of Theorem 1 on the unimodality of the functions $Q_j^-(L)$*and* $Q_j^+(L)$ Therefore, Theorem 1 is proven.

Corollary 1. Let L_j^- *and* L_j^+ be the maximum points of the corresponding functions Q_j^- *and* $Q_j^+, j = 1,\ldots,n$. Then the value L^*— the point of the global maximum of the function $D(L)$ — satisfies the condition:$L_1^* \leq L^* \leq L_2^*$ *where* $L_1^* = \min\{L_j^-, L_j^+, j = 1,\ldots,n\}$; $L_2^* = \max\{L_j^-, L_j^+, j = 1,\ldots,n\}$.

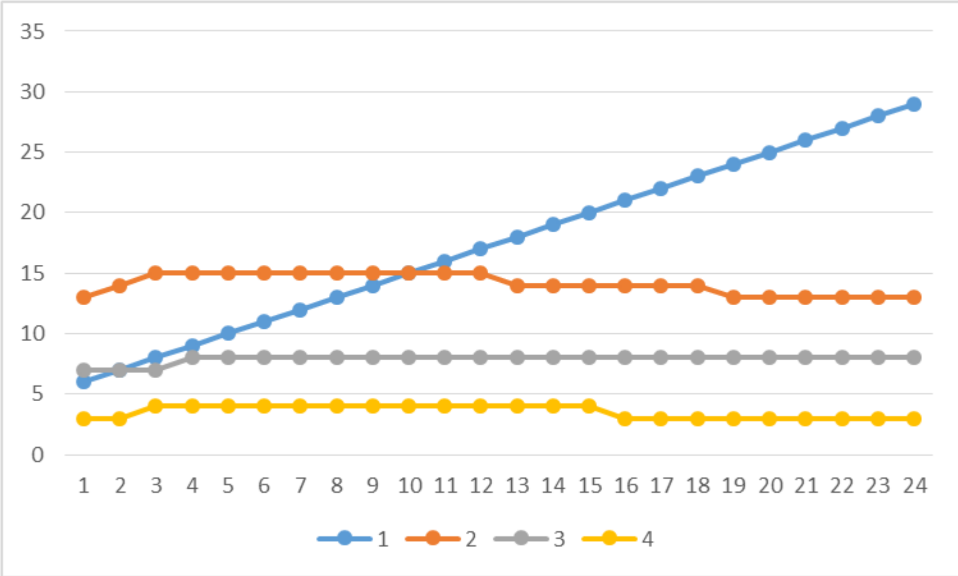

Figure 1. Specific income per unit of time

Dependence of $F_j(L)$ *on* L: 1 – Q(L); 2 – $F_1(L)$; 3 – $F_2(L) = F_3(L)$; 4 – maximum values of functions $F_j(L)$ *and* $Q(L)$ in particular, with the same $j = 1,\dots,n$ values of μ_j, λ_j, s_j *and* a_j the function $Q(L)$ is unimodal. The Figure 1 shows graphs of the functions $Q(L) and\ F_j = \left(1 - \frac{\lambda_j}{\lambda}\right) Q_j^+(L) = \left(\frac{\lambda_j}{\lambda}\right) Q_j^-(L), j = 1,2,3,$ demonstrating the results of Theorem 1, with the following parameters: $\lambda_1 = 4$; $\lambda_2 = \lambda_3 = 4/3; \mu = 3$; $s_1 = s_2 = s_3 = 3$; $C_0 = 21$; $C_{0,j} = 21; C_{1,j} = 11$; $C_{2,j} = 4$; $C_3 = 1$; $C_4 = 0{,}02$.

D. Conclusion

The main result of the article is the proof of the statement that the income function of the considered is a linear combination of unimodal functions. The practical importance of the result lies in the fact that for such a function the boundaries of the region where the global maximum point is guaranteed to lie are the minimum and maximum values on the set of global maxima of the indicated unimodal functions.

For the considered similarly to Theorem 1, the following statement is also proved: in the case of a scheme (with $L = 0$) if N is a fixed quantity and a_j are variable quantities, $j = 1,\dots,n$, then the income function is the sum $\sum_{j=1}^{n} f_j(a_j)$ of unimodal functions $f_j(a_j)$ on the interval $[1, N]$ such that $f_j(a_j)$ is convex in a_j on the corresponding interval $[1, a_j^*]$, *where* $[\ a_j^*, \dots, a_n^*)$ is the global maximum point of the income function on the set of sets $(a_1, \dots, a_n), \sum_{j=1}^{n} a_j \le N, a_j > 0, j = 1,\dots,n$. From this statement follows a simple rule for searching for the global maximum point of the income function in the scheme: while $\sum_{j=1}^{n} a_j < N$ we increase successively at each step by 1 one of the variables a_j with index j^* such that the increment

$$f_{j*}(a_{j*} + 1) - f_{j*}(a_{j*}) = \max_j\{ f_j(a_j + 1) - f_j(a_j) > 0, j = 1,\dots,n\}.$$

The results of the work can be used in the development and operation of information and production streaming systems with limited capacity storage devices to improve their efficiency.

E. References

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